

FLUENCY FOR ALL



How To Fast Track Your Students to Fact Mastery

Addition & Subtraction

FACT FLUENCY MATTERS

More and more, teachers are telling us that their students don't know their addition and subtraction facts. In a world where *Siri* can give you the answer to $8 + 6$ in seconds and critical and divergent thinking are the "new basics", does fact mastery really matter anymore?

THE HOOK:

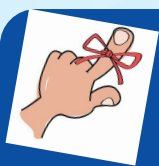
To make something more memorable reduce what needs to be learned. StrADdegY connects the 121 disparate addition facts to shared structures, patterns and relationships that are then mapped to five strategies.



We believe it does, not because automaticity is a proxy for what it means to be numerate, but rather, fact fluency frees children's cognitive processing for more important mathematical discovery. When children's short-term memory is consumed by the need to calculate basic single-digit computations they don't have the stamina for more complex problem solving. In the process, they begin to see themselves as failures in math, a self-imposed label that becomes needlessly debilitating.

Why are facts so difficult for children to commit to memory? Too often they are treated in isolation, without attention to structure and relationships and without sufficient purposeful practice embedded in meaningful contexts.

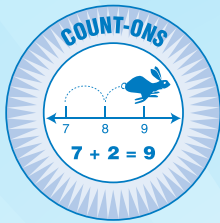
We feel that the question is not whether or not facts should be taught, but rather how they should be taught. To learn basic addition and subtraction facts don't focus on 121 discrete facts, cluster them according to number relationships and patterns that provide the bedrock for strategies. If facts are to be truly mastered, they must be presented in problem solving contexts that invite children to construct the number patterns, relationships and structures that lead to a full understanding of the composition and decomposition of number.



Please Remember:

Students only attend to strategies when they are ready, when the underlying big ideas upon which they are built are fully internalized. Just because the child has been exposed to (taught) a strategy is absolutely no guarantee that s/he is ready to use it. As you work through this framework make note of the big ideas and student thinking that is required for a child to use a strategy with intention.

What follows is a framework and cadence designed to move children away from a reliance on inefficient counting strategies and rote memorization to a mastery of addition and subtraction facts constructed from an understanding of the parts of numbers and their relationships:



THE STRATEGIES

Count-ons come first in the sequence. They come naturally to children and the impact on student confidence is significant. With very little effort, children realize that they can use a simple number relationship (counting on/counting back 1 or 2) to master 57 of the 121 facts.

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
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7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20



The doubles facts involve addends that are the same and come very quickly. They become the foundation to derive the surrounding facts: the doubles plus or minus one or two.

Just the Facts

There are 11 doubles facts; only 8 are new (doubles 0, 1, 2 were explored previously). These 11 facts become the foundation to build another 38 Facts known as near-doubles. Of these 38 facts 12 were already explored as a Count-On and 13 of the remaining 26 facts are "turn-arounds". Let's celebrate! With two easily accessible strategies (Count-Ons and Think Doubles) 91 of the 121 facts have been mastered, only 30 more to go.

Big Ideas:

Hierarchical Inclusion:

Get it: I know that 6 is one less than 7.

Use it: If 6 is nested in 7, then I can think of $6 + 7$ as $6 + 6$ and 1 more.

Associative Property: Using Compensation and Equivalence:

Get it: When I count, the sequence is 6, 7, 8. I know that 7 falls in the middle, one away from both 6 and 8.

Use it: When I think of $8 + 6$, I know that 7 is both 1 more than 6 and 1 less than 8. I can take 1 from the 8 and give it to the 6 to turn the problem into a double, $7 + 7$.

Just the Facts

As children reflect on the power of the commutative property they realize that there are only 30 distinct Count-On facts that need to be remembered. The remaining 27 facts are "turn-arounds". For example: $2 + 7$ is the same as $7 + 2$.

Big Ideas:

Cardinality:

Get it: I count a collection of objects to find out how many there are. I don't have to count a second time. As long as I don't add more or take any away, the total will always be the same.

Use it: After counting, if I add more objects to the collection, I don't need to count the first group again. I can just keep counting on from where I finished.

Hierarchical Inclusion:

Get it: When I count, the number I say next is always one more than the number that came before.

Use it: If 4 is always 1 less than 5, then $4 + 1$ must always be 5 and $5 - 1 = 4$.

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
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7	7	8	9	10	11	12	13	14	15	16	17
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THE STRATEGIES

Up next are the Bonds to Ten. These facts are clustered around a shared relationship more than a common strategy. They all represent part-whole combinations to ten, the foundational building block of our number system. Bonds to Ten must be mastered early.

Just the Facts

There are only 11 facts in this cluster, 9 of which have been targeted in previous strategies and the two remaining facts are commutative, making things even easier: $7 + 3$, $3 + 7$. This cluster still receives full attention given how important number combinations to ten are to student's flexible thinking as they progress through the grades.

Big Ideas:

Conceptual Subitizing:

Get it: I can look at a collection and see smaller groups within it.

Use it: I can break up a whole into smaller parts and recombine them to find out "how many"

Subtraction as Missing Part:

Get it: I can break up a whole into one or more parts.

Use it: If I know the whole and one of the parts, I can figure out the missing part by thinking about a relationship I already know.

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7	7	8	9	10	11	12	13	14	15	16	17
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10	10	11	12	13	14	15	16	17	18	19	20



The facts in this cluster are traditionally not considered basic facts, but we feel they deserve full exploration to reveal that "teen" numbers are simply a special decomposition of number where one part is a ten (ten ones) and the other part is a collection of additional ones. This critical understanding of early place value remains a mystery to many young learners for far too long. It requires early and consistent exploration.

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
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Just the Facts

There are 21 facts in this cluster, 11 of which have been targeted in previous strategies. Of the remaining 10 facts 5 are commutative leaving only $3 + 10$, $4 + 10$, $5 + 10$, $6 + 10$, $7 + 10$ and their "turn-arounds" to explore meaningfully to automaticity.

Big Ideas:

Part-Whole Relationships (early unitizing):

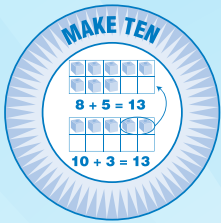
Get it: When I count 10 items I can think of them as 10 units or 1 ten.

Use it: I can break up teen numbers in any way I like, but one way that can be helpful is to think of the teen number as one ten and some more ones. 17 is $10 + 7$.

Taking From Ten (early place value):

Get it: I can group collections greater than 10 as a group of 10 ones and some more ones.

Use it: When I take a number away from a teen number, I can think of the teen number as one ten and some more ones. $17 - 9$. 17 is $10 + 7$. I can take the 9 from the 10 and add the 1 that is leftover to 7 to make 8.



THE STRATEGIES

Our journey through four strategies has brought us close to full fact fluency. The final strategy combines what children know about compositions of numbers from 1 – 9 (**Bonds to Ten**) and what happens when ten is added to any quantity from 1 – 9 (**Ten and Some More**) to make sense of all but two of the remaining facts (6 + 3 and 3 + 6). The **Make Ten** strategy can be used when one of the addends is a 7, 8, or 9 and the other addend is large enough to cause the sum to bridge the decade. For example, to solve $9 + 6$, children use their understanding of the number bonds from 1 to 10 to decompose 6 into 5 and 1. The 1 is combined with the 9 to make 10. The 5 that remains is added to the 10 to make 15 (**Ten and Some More**). $9 + 6$ has been changed to $10 + 5$, a much simpler computation.

+	0	1	2	3	4	5	6	7	8	9	10
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9	9	10	11	12	13	14	15	16	17	18	19
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Just the Facts

There are 33 facts in this cluster. 17 have been captured within other strategies. Half of the remaining 16 facts are “turn arounds” leaving only 8 combinations to really target intensively.

Big Ideas:

Equivalence - using compensation and the associative property

Get it: It doesn't matter how I break up the whole or how I recombine the parts, as long as I don't add any new items or take any away, the total stays the same.

Use it: When I am adding a 7, 8 or 9 to another number and the total is greater than 10, it might help me to break up the other number into parts. I can combine one of the parts to the 7, 8 or 9 to make ten and then add on the rest. $8 + 6$ is the same as $8 + 2 + 4$. $8 + 2$ makes 10 and 4 more is 14.

Our journey through this strategic framework is complete. It has brought us to a wonderful destination of fact mastery. We chose a path firmly rooted in conceptual understanding and avoided the narrower passage of rote memorization. Along the way children developed a fundamental belief that with practice, persistence and flexible thinking difficult mathematical challenges can be overcome. The resulting confidence will pay dividends for years to come.

This framework helps you know where you are going and why you want to get there. If you would like more help navigating the route, please feel free to visit www.strADdegY.com to access a **FREE TRIAL** of our teacher portal full of 100's of games, story contexts, number strings, digital tools, and problems to build **Facts that Last!**

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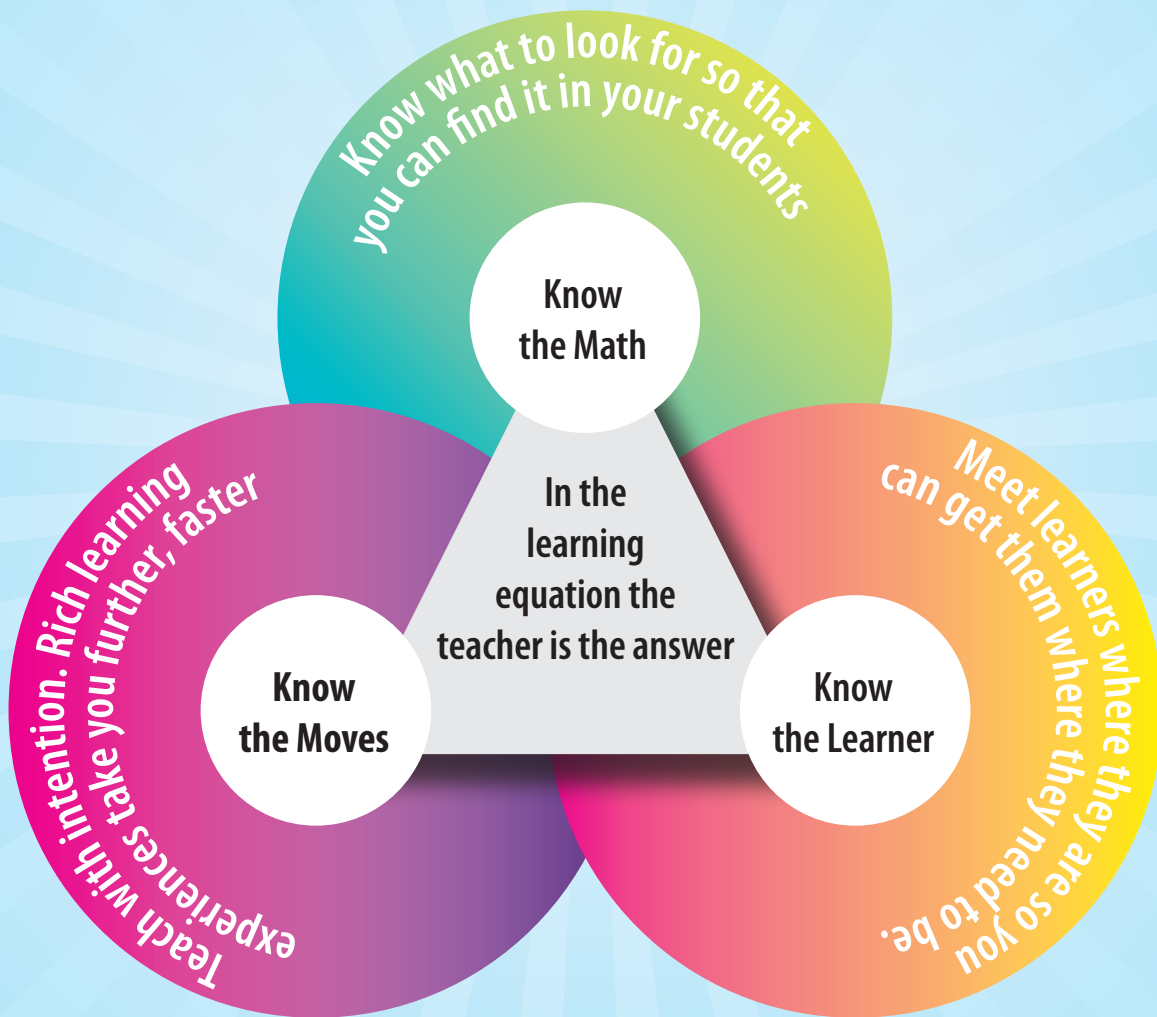
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FACTS THAT LAST



Memory Through Meaning K - 2

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